SUPPLEMENTARY PROBLEMS FOR CHAPTER 5

1. A linear shift invariant system is described by the difference equation

$$y[n] = 0.3y[n-1] + x[n] + 1.2x[n-1]$$

   (a) Is the system minimum-phase? Tell why or why not.

   (b) Write a pair of difference equations involving the auto- and cross-correlation functions which when solved would allow you to find $R_y[l]$ if you knew $R_x[l]$.

   (c) Solve the difference equations for $R_y[l]$ assuming the input is binary white noise with variance $\sigma_x^2 = 2$.

   (d) If the input is the white noise process in part (c), what is the complex spectral density function of the output $S_y(z)$?

   (e) Solve the given difference equation for the system impulse response and by convolution find $R_y[l]$. Transform $R_y[l]$ to find $S_y(z)$ and check with your answer to part (d).

2. Beginning with the diagram of Fig. 5.2, and without referring to the table, reconstruct Table 5.1 on page 238.

3. Tell which of the following are legitimate complex power spectral density functions.

$$\text{(i)} \quad S_x(z) = \frac{z - 4 + z^{-1}}{2z - 5 + 2z^{-1}}$$

$$\text{(ii)} \quad S_x(z) = \frac{2z + 1 + 2z^{-1}}{-2z + 5 - 2z^{-1}}$$

   Hint: Consider the innovations representation.

4. A random process $x[n]$ with correlation function

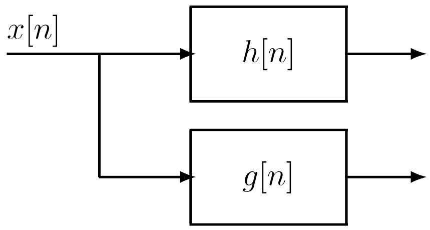
$$R_x[l] = \tfrac{4}{3}\left(\tfrac{1}{2}\right)^{|l|}$$

   is applied to a real unknown linear shift-invariant system. The output of the system is another random process $y[n]$ with complex spectral density function

$$S_y(z) = \frac{0.2z - 1.04 + 0.2z^{-1}}{0.5z - 1.25 + 0.5z^{-1}}$$

What is the system function $H(z)$ of the system? (Assume that the system is minimum-phase.)

5. A discrete random signal $x[n]$ is input simultaneously to two linear shift-invariant systems with impulse responses $h[n]$ and $g[n]$.



   (a) Find an expression for the cross-correlation function between the two outputs in terms of the input correlation function $R_x[l]$ and the impulse responses of the two systems. Express this in terms of simple convolution operations.

   (b) Find the corresponding expression for the complex cross-spectral density function for the two outputs in terms of $H(z)$, $G(z)$, and $S_x(z)$. Evaluate this at $z = e^{\jmath\omega}$ to obtain an expression for the cross-spectral density function.

6. A *real* time-invariant system is described by the difference equation

$$y[n] = ay[n-1] + x[n]$$

   where x is the input and y is the output.

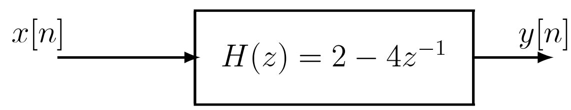
   (a) What is the cross-correlation function $R_{yx}[l]$ between the output and the input when the input is white noise with variance σ_o^2?

   (b) If the input is a Bernoulli process with $\mathrm{P} = \frac{1}{2}$, what is the complex spectral density function $S_y(z)$?

7. A random process $x[n]$ with complex spectral density function

$$S_x(z) = \frac{-\sqrt{3}}{z - 4/\sqrt{3} + z^{-1}}$$

is passed through the linear shift-invariant filter shown below:



(a) Find the complex spectral density function of the filter output $S_y(z)$.

(b) Find and draw the innovations representation for $y[n]$.

8. Consider the random processes $y[n]$ and $x[n]$ defined in Prob. 7 of Supplementary Problems for Chapter 4.

   (a) Represent the relation between these processes as a linear shift-invariant filter and draw the block diagram. What is the filter transfer function $H(z)$?

   (b) Using this result, compute the power spectral density function and the complex spectral density function for the process $y[n]$. Express your answers in the simplest possible terms.

9. A random process has the complex spectral density function

$$S_x(z) = -12z + 25 - 12z^{-1}$$

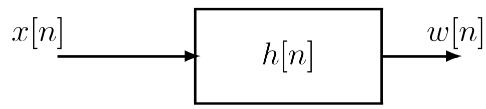
   (a) Find and draw the innovations representation for the process.

   (b) Find a *causal stable filter* which if applied to the random process, would convert it to white noise. What is the white noise variance?

10. A zero-mean random process $x[n]$ has a correlation function given by

$$R_x[l] = (0.5)^{|l|}$$

Find the impulse response $h[n]$ of the minimum-phase filter shown below that transforms the input into a white noise process $w[n]$ with variance $\sigma_w^2 = 1$.



11. White noise with variance $\sigma_w^2 = 1$ is passed through a linear system with impulse response

$$h[n] = \tfrac{1}{2}\left(\delta[n] - \delta[n-1]\right)$$

   (a) What is the autocorrelation function of the output?

   (b) What is the power spectral density function of the output?

12. **(a)** Draw the *innovations representation* for the random process $x[n]$ with complex spectral density function

$$S_x(z) = \frac{7}{-12z + 25 - 12z^{-1}}$$

   (b) What is the filter $H(z)$ that would convert $x[n]$ to white noise?